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Rayleigh-Schrödinger perturbation theory for electron-phonon interaction in two dimensional quantum dots with asymmetric parabolic potential

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Abstract. Within the framework of the second-order Rayleigh-Schrödinger perturbation theory, we investigate the effects of the interaction of the electron and longitudinal-optical phonons in two-dimensional semiconductive quantum dots with respect to a general potential. We propose a simple expression for the ground state energy, and compare it with those obtained by Landau-Pekar strong coupling theory. It is shown both analytically and numerically that the results obtained from the second-order Rayleigh-Schrödinger perturbation theory could be better than those from Landau-Pekar strong coupling theory when the coupling constant is sufficiently small. Moreover, some interesting problems, such as polarons in quasi-one-dimensional quantum wires, and quasi-zero-dimensional asymmetric or symmetric quantum dots can be easily discussed only by taking different limits. After the numerical calculations, we find that there exists a simple dimensional scaling and symmetry relation for the ground state polaron energy. Furthermore, we apply our results to some weak-coupling polar semiconductors such as GaAs, CdS. It is shown that the polaronic effects are found to be quiet appreciable if the confinement lengths and smaller than a few nanometers.

PACS. 71.38.+i Polarons and electron phonon interactions – 63.20.Kr Phonon electron and phonon phonon interactions

1 Introduction

With the recent development in micro-fabrication technology, such as molecular-beam and lithographic deposition, it has created a variety of opportunities for the fabrication of synthetic semiconductor structures with reduced dimensionality [1–8]. The effects of electron optical phonon interaction on the energy levels, effective mass and the polaronic properties of low dimensional confined electrons have attached much attention for their potential applications and a lot of new interesting phenomenon [9–27]. It is shown in these investigations that polarons in low dimensional quantum structures are remarkably different from those in bulk material due to the presence of different confining potentials, which may cause the different confinement for the carriers motion. However, many of them [10–13, 23, 24] are always refrained from including the coupling of the electron to the confined phonon modes as well as bulk-phonon approximation, thus they can give a clear view of the bulk-phonon effects. Moreover, the electron-phonon coupling is rather weak in actual material of interest, say, GaAs ($\alpha = 0.07$), then some theory like LP strong-coupling theory *etc.*, are unsuitable for dealing with such problems. Because Rayleigh-Schrödinger perturbation theory (RSPT) were usually applied efficiently in weak-coupling field. Then it would be helpful for a better understanding of the role of electron-LO-phonon interaction in quantum dots, if we devote RSPT to handle such problems. Meanwhile, RSPT can always show the simple form of the electron-phonon interaction and deduce some simple closed-form analytical expressions for the polaronic correction to the ground state energy. For example, Degani and Farias [32] have ever used RSPT to calculate the polaronic correction to the self-energy and the effective mass of the electron confined in a symmetric parabolic quantum dot. In addition, Mukhopadhyay and Chatterjee [11] have derived an analytical expression for the second-order RSPT correction to the self-energy and obtained a scaling relation for the ground state energy of an electron in multi-dimensional symmetric quantum dot. Thus, the second-order RSPT is imperative to provided some qualitative insight into the investigation on polarons in confined media made from polar crystals.

The purpose of this paper is to give a more advanced investigation of the properties for two-dimensional (2D) asymmetric quantum structures. We consider here the same model polaron problem in a three-dimensional

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(3D) quantum structure, but the electrons are much more strongly confined in one direction than in the other two directions. As an interesting theoretical model [37–39], we assume that the confining potential in a single quantum dot is parabolic and different in different directions of x and y. Alternatively, we point out here that there exist some simple expression for the polaron energy correction corresponding to the different quantum structures. We can unify all these cases in a three-dimensional figure. Moreover, we may compare them with some results obtained from experiment and LP strong-coupling theory, and then we will give a clear view of solely the low dimension effects of electron optical phonon interaction within the framework of the second-order RSPT. We provided a broad interpolating overview to the two-dimensional polaron problem consisting of an electron perfectly confined in a boundary with symmetric or asymmetric parabolic potential.

This paper is organized as follow. In Section 2, we derive the energy expressions for polarons in partly symmetric and entirely asymmetric potential within the framework of the second-order RSPT and compare those from LP strong-coupling theory. In Section 3, we present our numerical results over reasonably wide ranges of the confinement potential, compare for some materials to other important energy scales, such as the exciton binding energy. At last, we give out a summary of this paper.

2 Formulation

We start with three-dimensional Fröhlich Hamilton for an electron moving in an asymmetric low dimensional quantum structure and interacting with the LO phonons.

$$H = -\frac{1}{2}\nabla_r^2 + \frac{1}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right) + \sum_q a_q^+ a_q + \sum_q \left(\xi_q \exp\left(-i\mathbf{q}\cdot\mathbf{r}\right) a_q^+ + h.c.\right], \quad (1)$$

where all vectors are three dimensional and the units have been chosen such as $\hbar = m = \omega_0 = 1$ (Feynman units), ω_0, the optical phonon frequency, is assumed to be dimensionless, $\mathbf{r}$ refers to the position vector of the electron, $\omega_x = \omega_{hx}/\omega_0$, $\omega_y = \omega_{hy}/\omega_0$, $\omega_z = \omega_{hz}/\omega_0$, ω_{hx}, ω_{hy} and ω_{hz}, respectively, measure the confining strength of the parabolic potential for directions x, y and z, a_q^+ and a_q are the creation and annihilation operators for a LO phonon of wave vector $\mathbf{q}$, and ξ_q for the 3D systems is always has [34]

$$|\xi_q|^2 = \frac{2^{3/2}\pi}{\nu q^2}\alpha, \quad (2)$$

where ν is the volume of the 3D quantum structure and α is the electron-phonon coupling constant. In addition, for most low dimensional quantum structure, the value of α is small, so the weak coupling approximation should be valid.

Then performing the second-order RSPT correction to the ground-state (GS) electron self-energy for the polaronic interaction, we have

$$\Delta E = -\sum_j \sum_q \frac{\left|\langle \phi_j(r)\left|\xi_q \exp\left(-i\mathbf{q}\cdot\mathbf{r}\right)\right| \phi_0(r)\rangle\right|^2}{E_j - E_0 + 1}, \quad (3)$$

with

$$\left\{-\frac{1}{2}\nabla_r^2 + \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{2}\omega_z^2 z^2\right\}\phi_j(\mathbf{r}) = E_j \phi_j(\mathbf{r}), \quad (4)$$

$$\phi_j(\mathbf{r}) = \left(\frac{\omega_x^{1/2}\omega_z^{1/2}\omega_y^{1/2}}{\pi^{3/2}2^{j_1+j_2+j_3} j_1! j_2! j_3!}\right)^{1/2} H_{j_1}(\sqrt{\omega_x}x)\, H_{j_2}(\sqrt{\omega_y}y) \times H_{j_3}(\sqrt{\omega_z}z) \exp\left[-\frac{1}{2}\omega_x x^2 - \frac{1}{2}\omega_y y^2 - \frac{1}{2}\omega_z z^2\right], \quad (5)$$

$$E_j = \left(j_1 + \frac{1}{2}\right)\omega_x + \left(j_2 + \frac{1}{2}\right)\omega_y + \left(j_3 + \frac{1}{2}\right)\omega_z, \quad (6)$$

$H_n(\omega x)$ being the Hermite polynomial of order n. This expression is just as same as which occurred in the variational ground state energy of Coulomb impurity-bound polaron in the Feynman-Haken path integral calculation with a harmonic oscillator effective trial action [28–31].

Using the transformation

$$\frac{1}{E_j - E_0 + 1} = \int_0^\infty \exp\Big[-\left(E_j - E_0 + 1\right)t\Big]\, dt, \quad (7)$$

and the Slater sum rule for the Hermite polynomials

$$\sum_n \frac{1}{2^n n!} H_n(\lambda x) H_n(\lambda x') \exp\left[-\frac{1}{2}\lambda^2(x^2 + x'^2) - 2np\right] = \frac{\exp(p)}{\sqrt{2\sinh(2p)}} \exp\left\{\left(-\frac{1}{4}\lambda^2\right)\left[(x+x')^2 \tanh p + (x-x')^2 \coth p\right]\right\}. \quad (8)$$

We can easily perform the summation over j_x, j_y and j_z in (3) respectively. Then using

$$\sum_q \frac{\exp\left[i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')\right]}{q^2} = \frac{v}{4\pi}\cdot\frac{1}{|\mathbf{r}-\mathbf{r}'|}, \quad (9)$$

One can integrate with respect to the electron position vectors $\mathbf{r}$ and $\mathbf{r}'$ transforming these vectors into two new variables $\mathbf{u}$ and $\mathbf{v}$ as $\mathbf{u} = \frac{1}{2}(\mathbf{r}+\mathbf{r}')$, $\mathbf{v} = \frac{1}{2}(\mathbf{r}-\mathbf{r}')$.

Then the polaron self-energy correction can be show as follows

$$\Delta E = -\frac{\alpha\omega_x\omega_y\omega_z}{\pi^3}\int_0^\infty dt\cdot\frac{2\sqrt{2}\cdot e^{-t}}{\sqrt{\left(1-e^{-\omega_x t}\right)\left(1-e^{-\omega_y t}\right)\left(1-e^{-\omega_z t}\right)}} \times \int \frac{1}{|\mathbf{v}|}\cdot e^{-\left(v_x^2 D_x + v_y^2 D_y + v_z^2 D_z\right)}\cdot d\mathbf{v}\cdot\int e^{-\left(u_x^2 c_x + u_y^2 c_y + u_z^2 c_z\right)}\cdot d\mathbf{u}, \quad (10)$$

where

$$c_i = \omega_i \left[1 + th\left(\frac{1}{2}\omega_i t\right)\right],$$
$$D_i = \omega_i \left[1 + \coth\left(\frac{1}{2}\omega_i t\right)\right] \cdot (i = x, y, z).$$

If the electrons are much more strongly confined in one direction (taken as the z direction) than in the other two directions, what we discussed is a very interesting theoretical model, namely the 2D asymmetric quantum dot. When we take the limit $\omega_z \to \infty$, then $\lambda_z \to \infty$. So we can deal with the integral for electron coordinates u_z and v_z firstly, by introducing the following transformation

$$\lim_{a\to\infty}\left(\int_{-\infty}^{+\infty} e^{-ax^2}\cdot f(x)\,dx\right) = \lim_{a\to\infty}\left(\int_{-\infty}^{+\infty}\delta(x)\cdot\frac{\sqrt{\pi}}{\sqrt{a}}\cdot f(x)\,dx\right) = \lim_{a\to\infty}\left(\frac{\sqrt{\pi}}{\sqrt{a}}\cdot f(0)\right) \tag{11}$$

and then integrating with respect to the remain electron coordinate u_x, u_y, v_x, and v_y. Finally we have

$$\Delta E = -\frac{\alpha\sqrt{\omega_y}}{\pi^{1/2}}\int_0^{\infty} dt \frac{e^{-t}}{\sqrt{(1-e^{-\omega_y t})}}\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-(1-x)\sin^2\theta}}, \tag{12}$$

where

$$x = \frac{\omega_y\left[1+\coth\left(\frac{1}{2}\omega_y t\right)\right]}{\omega_x\left[1+\coth\left(\frac{1}{2}\omega_x t\right)\right]}\cdot$$

The properties of 2D asymmetric quantum dots in which only the motion of the electrons in the x-y plane is taken into account, can be described perfectly by the above equation. More interestingly, from equation (12) only by selecting the different terms, we can discuss all kinds of cases included in 2D semiconductor quantum structures. There, we can find the further results of this integral equation are determined by the relative scale between $\omega_x\left[1+\coth\left(\frac{1}{2}\omega_x t\right)\right]$ and $\omega_y\left[1+\coth\left(\frac{1}{2}\omega_y t\right)\right]$. Meanwhile, since the increase of x is always accompanied by a progressive increase in function $f(x) = x\left[1+\coth\left(\frac{1}{2}xt\right)\right]$. Then, the relation between $\omega_x\left[1+\coth\left(\frac{1}{2}\omega_x t\right)\right]$ and $\omega_y\left[1+\coth\left(\frac{1}{2}\omega_y t\right)\right]$ is just corresponding to that between the relative values of ω_x and ω_y. If we change the relative values of ω_x and ω_y, some different forms would be obtained. In general, there exist three different types of low-dimensional semiconductor quantum structures, such as the symmetric and asymmetric quantum dots, quantum wires, included in the case of $\omega_x = \omega_y$, $\omega_x > \omega_y$.

With $\omega_x = \omega_y = \omega$, we might get the second-order RSPT correction for the polaron self-energy like following form by equation (12),

$$\Delta E = -\frac{\pi}{2}\cdot\frac{\alpha}{\sqrt{\omega}}\cdot\frac{\Gamma\left(\frac{1}{\omega}\right)}{\Gamma\left(\frac{1}{\omega}+\frac{1}{2}\right)}, \tag{13}$$

selecting $l = 1/\sqrt{\omega}$ and rewriting the above equation, we might have

$$\Delta E = -\frac{\pi\alpha l}{2}\cdot\frac{\Gamma\left(l^2\right)}{\Gamma\left(l^2+\frac{1}{2}\right)}\cdot \tag{14}$$

l is the dimensionless confinement length and given by $l = \frac{l_0}{r_0}$, where l_0 and r_0 are defined as $l_0 = \left(\hbar/(m\omega_h)\right)^{1/2}$ and $r_0 = \left(\hbar/(m\omega_0)\right)^{1/2}$. It is just the case of 2D symmetric quantum dots, discussed by Mukhopadhyay, Chatterjee [11], and the others [33,34]. Thus, the results are naturally as same as theirs.

On the other hand, it may be noted here that if the confinement of direction x is larger than that of direction y, we may obtain the case of 2D asymmetric quantum dots as follows.

$$\Delta E = -\frac{\alpha\sqrt{\omega_y}}{\pi^{1/2}}\int_0^{\infty} dt\cdot\frac{e^{-t}}{\sqrt{(1-e^{-\omega_y t})}}\cdot F(x), \tag{15}$$

with

$$F(x) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-(1-x)\sin^2\theta}} \qquad (0 < x \le 1),$$

the first type elliptic integral. This equation obviously yields the results of 2D quantum dots with asymmetric parabolic potential as well, where the confinement for direction x is stronger than that of direction y. Subsequently, we will discuss the other important cases, which are deduced by taking some different limits.

From the expression for 2D asymmetric quantum dots (Eq. (12)), we find, under $\omega_z \to \infty$, if the confinement of direction y is in the weak confinement limit (*i.e.* $\omega_y \to 0$), the above equation will obviously yield the case of asymmetric quantum wires. Here we have included the following limits into: $\lim_{\omega_y\to 0}\omega_y\left(1+\coth\frac{1}{2}\omega_y t\right) = \frac{2}{t}$, $\lim_{\omega_y\to 0}\left(1+\tanh\frac{1}{2}\omega_y t\right) = 1$, and $\lim_{\omega_y\to 0}\frac{\omega_y}{sh\omega_y t} = \frac{1}{t}$. One can easily show the form of polaron GS energy correction as follows

$$\Delta E = -\frac{\alpha}{\sqrt{\pi}}\int_0^{\infty} dt\cdot\frac{e^{-t}}{\sqrt{t}}\cdot F(x'), \tag{16}$$

where $x' = 2/\left[t\cdot\omega_x\left(1+\coth\frac{1}{2}\omega_x t\right)\right]$. This formula is just related to the case of quasi-one dimensional asymmetric quantum wires and it is sure to show the nature of this system.

Next, we will continue to discuss another case corresponding to the different limit of equation (13). After investigating all kinds of cases that possibly exist, we might find: if we include the weak-confinement for direction x as same as the another direction y, the form then corresponds to the ground state energy of pure 2D free polaron ($\Delta E = -\frac{\pi}{2}\alpha$). This result is also similar to what reduced by LLP variational calculations [35,36], Feynman path integral method [28–31] and second-order perturbative theory [23] *etc.*

According to the above discussion, we would find something interesting that: if the effective confinement potential satisfy: $\omega_x \to \omega$, $\omega_y \to \infty$, the corresponding model is 1D polaron, which is always diverging. It just corresponds to the divergence of the interaction coefficients [33]:

$$\lim_{N\to 1} |\xi_q|^2 = \lim_{N\to 1} \frac{\Gamma\left[\frac{1}{2}(N-1)\right] 2^{N-(3/2)} \pi^{(N-1)/2}}{v_N q^{N-1}} \alpha \to \infty \tag{17}$$

so, it usually needed to be dealt with by the renormalization of the coupling constant α.

Moreover, for comparison, we will study these structures within LP varitional theory as well. The strong-coupling polarons in quantum dots can also be investigated by LP variational scheme [34]. Alternatively, we here proceed to give a more concise representation of this scheme. The adiabatic polaron ground-state can be given through following product ansatz

$$\begin{aligned} |\ldots\rangle &= \phi(\mathbf{r})|\mathbf{A}\rangle \\ &= \frac{(\lambda_x \lambda_y)^{1/2}}{\sqrt{\pi}} \cdot e^{-\left(\lambda_x^2 x^2 - \lambda_y^2 y^2\right)/2} \cdot e^{\left(\sum_q f_q b_q^+ - h.c.\right)} |0\rangle, \end{aligned} \tag{18}$$

where $|0\rangle$ is the unperturbed zero phonon state satisfying for all $\mathbf{k}$, λ_x and λ_y are the variational parameters, as $f(\mathbf{k})$, $f^*(\mathbf{k})$, they will be determined variationally.

To find the optimal fit to $f(\mathbf{k})$, and $f^*(\mathbf{k})$, we should minimize the expectation value of the Hamiltonian (1) in the above trial state $\langle \ldots |H| \ldots \rangle$, which has the following functional form

$$\begin{aligned} E\big(f(\mathbf{k}), f^*(\mathbf{k})\big) &= \frac{1}{2}\left(\lambda_x^2 + \lambda_y^2\right) + \frac{\omega x^2}{2\lambda_x^2} + \frac{\omega y^2}{2\lambda_y^2} \\ &+ \sum_{\mathbf{k}} f^*(\mathbf{k}) f(\mathbf{k}) + \sum_{\mathbf{k}} v_{\mathbf{k}} e^{-\left(\frac{k_x^2}{4\lambda_x} + \frac{k_y^2}{4\lambda_y}\right)} f(\mathbf{k}) + h.c., \end{aligned} \tag{19}$$

and get

$$f(\mathbf{k}) = -v_{\mathbf{k}}^* e^{-\left(\frac{k_x^2}{4\lambda_x} + \frac{k_y^2}{4\lambda_y}\right)}, f^*(\mathbf{k}) = -v_{\mathbf{k}} e^{-\left(\frac{k_x^2}{4\lambda_x} + \frac{k_y^2}{4\lambda_y}\right)}. \tag{20}$$

Inserting these back into equation (19), we may have:

$$E^{LP} = \frac{1}{2}\lambda_x^2 + \frac{1}{2}\lambda_y^2 + \frac{\omega_x^2}{2\lambda_x^2} + \frac{\omega_y^2}{2\lambda_y^2} - \frac{\lambda_x \lambda_y \alpha \sqrt{\pi}}{\sqrt{\lambda_x^2 + \lambda_y^2}} \cdot \tag{21}$$

It is also expected that the second-order RSPT can produce good results for polarons in quantum dots with weak-coupling strength and confining potential. These will be demonstrated in the numerical calculations performed in the next section.

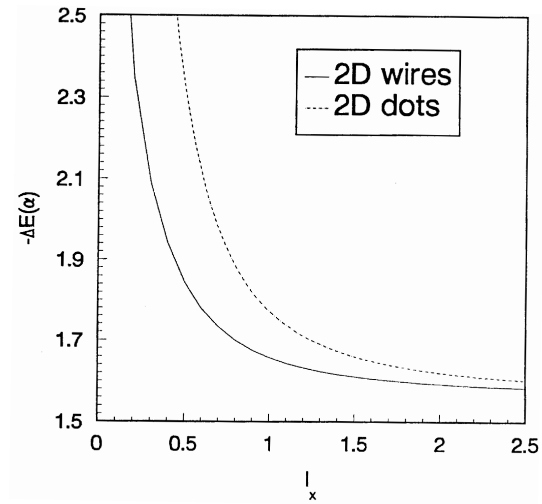

Fig. 1. Polaronic corrections $-\Delta E$ (Feynman units) to the GS energy of an electron in parabolic two dimensional symmetric quantum dots and asymmetric quantum wires as a function of confinement length l and l_x (Feynman units).

3 Numerical results and discussions

In order to calculate the polaronic energy correction, we should have the ground-state energy in the absence of electron phonon interaction. It is known that the ground-state energy to harmonic Hamiltonian $H = \mathbf{P}^2 + \frac{1}{2}\omega^2\mathbf{r}^2$ in N dimensions is given exactly by $E_{hm} = \frac{N}{2}\omega$. Hence, the polaronic correction to the ground-state energy in 2D quantum wires and 2D asymmetric quantum dots reads $\Delta E = E^{RSPT} - \frac{1}{2}\omega_x$, $\Delta E = E^{RSPT} - \frac{1}{2}\omega_x - \frac{1}{2}\omega_y$. As usual, the dimensionless confinement length of the semiconductor quantum structures is defined as $l_x = \frac{1}{\sqrt{\omega_x}}$, $l_y = \frac{1}{\sqrt{\omega_y}}$. Now, we will present some numerical results for the polaron in semiconductor quantum structures for arbitrary coupling constants and broad ranges of the confinement length of these structures by means of equations (12–16).

In Figure 1, we plot the polaronic energy correction ($-\Delta E$) to the ground state energy in both 2D symmetric quantum dots and quantum wires as a function of the confinement length l and l_x. Just as those gotten from experiment, it is clearly shown that the polaronic energy correction of quantum wires and quantum dots increase systematically with decreasing size and to increase in going from wires to dots of the same size. At the same time, the polaronic effect is substantially strengthened with contracting the confining length in both structures. The electron-optical phonon interaction has a pronounced effect on the electronic energy when the confinement length l (l_x) is sufficiently small. When the size of these two quantum systems increases, the polaronic correction increases and asymptotically assumes a constant value ($\Delta E = -\frac{\pi}{2}\alpha$), as stated by many other authors [11,23,28–31,35,36].

In Figure 2, we plot the polaronic correction ($-\Delta E$) to the ground state energy of 2D asymmetric quantum dots as a function of the confinement length l_x and l_y in

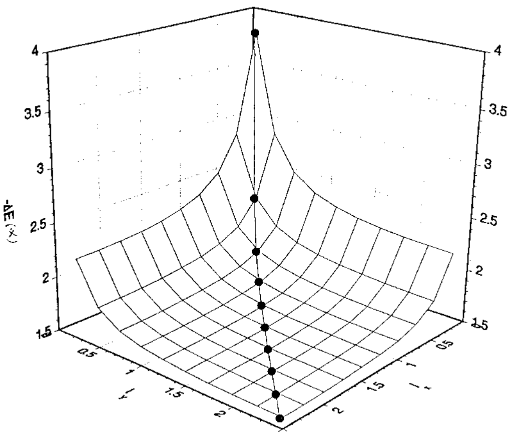

Fig. 2. Polaronic corrections $-\Delta E$ (Feynman units) to the GS energy of an electron in parabolic 2D dimensional asymmetric semiconductive quantum systems as a function of the confinement length l_x and l_y (Feynman units).

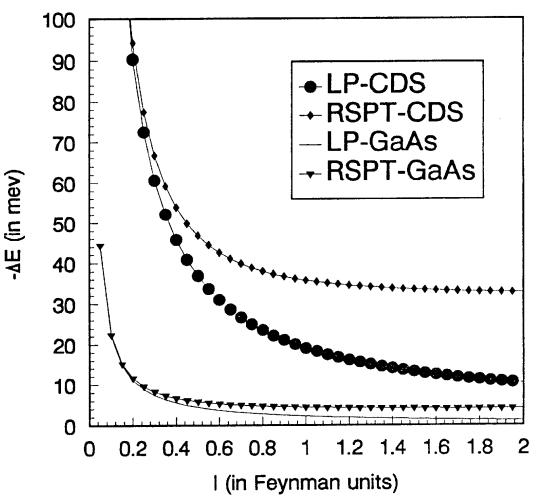

Fig. 3. Polaronic corrections $-\Delta E$ (in mev) to the GS energy of an electron in GaAs and CdS quantum dots with parabolic confinement in 2D, as a function of the confinement length l (Feynman units).

a 3D plot. It is observed that the electron-optical phonon interaction in asymmetric system has a more pronounced effect on the electronic energy than that of symmetric system, and the effect may be more remarkable with the increasing of asymmetry. Furthermore, the polaronic correction for electronic energy will become larger when the dot size is sufficiently small. Like the other two quantum series discussed above, the polaronic correction increased and asymptotically assumed a constant value as the size of dot increases. It also dedicates the essential features of 2D asymmetric semiconductive quantum systems. More interestingly, we can also observe the above two cases in this curved surface *i.e.* the curves at which the curved surface intersect with $l_x = 0$ (or $l_y = 0$) plane and $l_x = l_y$ plane are respectively corresponding to the results for asymmetric quantum wires and 2D symmetric quantum dots.

Table 1. Some parameters of CdS, and GaAs (ω_{LO} is in unit of meV and m in unit of bare electron mass).

Materials	m	ω_{LO}	α
CdS	0.155	38.26	0.527
GaAs	0.066	36.7	0.068

In Figure 3, we plot the variation of the polaron self-energy by the second-order RSPT and LP theory as a function of the confinement length for a few selected quantum dots of weak-coupling polar semiconductors such as GaAs, CdS. The material parameters used in the calculation are given in Table 1. It is evident from the figure that the polaronic effects increase quite considerably when the dot sizes are made smaller than a few nanometers. Figure 3 also shows that the results gotten from the second-order RSPT are far more efficient than those from LP theory, especially when the confinement length becomes larger. Then we can say that the weak-coupling approximation is justified for these low-dimensional structures. At large radii, however, the exact calculation shows that the magnitude of the shift of the ground state energy of GaAs (6 meV) is a little larger than that of exciton binding energy (< 5 meV) [40,41], then the shift of the ground state energy has to be considered in real materials as well. All of these results are very helpful in order to better understand the relevance and magnitude of the predicted effects for the actual structure.

4 Conclusions

In conclusion, we have investigated the polaron effect in the 2D asymmetric, symmetric quantum dots, and quasi-one dimensional quantum wires systems by the RSPT treatment. After considering all possible existed cases in 2D quantum structures, we can get the generality of such systems and the difference among these structures. Then, we unified them in a three-dimensional configuration. After the numerical calculations, it would be pointed out that for the same value of the electron-phonon-coupling constant α and the confinement length l, the polaronic effect is enhanced with lowering dimensionality and higher asymmetry.

It is shown that there exists some important relation between the polaronic effect and the size of the confinement. We can derive the relationship among these system from Figures 1 to 3 as follows. The polaronic effect of 2D symmetric quantum dots is weaker than asymmetric 2D-quantum dots, but stronger than that of quantum wires. With the size of confinement length and the symmetry increasing, the polaron self-energy correction for asymmetric 2D quantum dots, 2D symmetric quantum dots, and quasi-one dimensional quantum wires will asymptotically assume a same constant as large as $-\frac{\pi}{2}\alpha$. Meanwhile, we also can dedicate that the weak-coupling approximation is justified for real low-dimensional material structures, since the results gotten from the second-order RSPT are far more efficient than those from LP theory. Moreover, the exact calculation shows that the magnitude of the shift of the ground state energy of GaAs is a little larger than that of exciton binding energy.

Finally, it should be pointed out that the present theory is also suitable for the other more complicated problems. These extensions are in progress.

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